A Fully-Decoupled 3-DOF Translational Parallel Mechanism

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Abstract

A novel, revolutionary simple, 3-DOF translational parallel mechanism, called the Tripteron, is presented in this paper. The architecture of the Tripteron is such that its input-output equations are linear and fully decoupled – something that used to be thought impossible for a parallel mechanism. The obvious advantages of this architecture include a simplification of the kinematic computations, a constant diagonal Jacobian matrix, the absence of any singularities within the workspace, a rectangular box workspace, and mostly, a very intuitive kinematic behavior. First, the kinematics of the Tripteron is presented. Then, some important design issues are discussed. Finally, the mechanical design of the Tripteron is addressed and a working prototype is shown.

1 Introduction

After realizing in the late 1990s that industry needs low-cost 3-axis translational PKMs rather than complex hexapods, machine tool manufacturers have introduced quite a few of "tripods" and "triaxes." Most of them are based on the Delta robot concept /1/, including for example, the Triaglide built by Mikron, the Quickstep by Krause & Mauser, the VerticalLine V100 by INDEX-Werke, the Urane SX by Comau, and the PEGASUS by Reichenbacher. Several other similar PKMs were patented by Paul Sheldon /2/, the principal inventor of the Variax, who saw firsthand the need for 3-axis PKMs, after trying to market his hexapod. Several other translational PKMs, based on the use of a constraining chain, were also commercialized. Examples include the Ulyses built by Fatronik, and the SKM 400 by Heckert.

On the academic side, translational parallel mechanisms have been invented in great numbers, ever since Clavel proposed his Delta robot in 1990 /3/. Examples include the typical design with three universal-revolute-universal-joint legs or its slight variations /4, 5, 6/, or designs including parallelograms /7/, other than the obvious variations of the Delta robot. Several systematic approaches were also proposed for the type synthesis of translational parallel mechanisms, such as methods based on displacement group theory /8, 9/ and methods based on screw algebra or screw theory /10, 11/. However, these approaches deal mainly with the systematic type synthesis of (unactuated) translational parallel kinematic chains. As a result, the problem of selecting the joints to actuate was not examined systematically.

Although relatively simple, all of the above-mentioned translational parallel mechanisms have nonlinear coupled kinematics, singularities, a complex-shaped workspace, and a complex structural design. The principle of motion of any of these mechanisms can hardly be understood by anyone not familiar with mechanism kinematics.

In May 2001, however, a revolutionary simple 3-DOF translational mechanism, with fully decoupled linear input-output equations, was disclosed by the first two authors of this paper in a Canadian provisional patent application. An international PCT patent application was subsequently filed /12/ and a US patent is about to be granted (the application is already allowed). The mechanism, named the Tripteron, was found after a systematic investigation on the selection of the joints to be actuated. Later in 2002, Carricato and Parenti-Castelli /13/ and the first two authors of this paper /14/ proposed, separately, a large family of decoupled 3-DOF translational parallel mechanisms. At the same time, Kim and Tsai /15/ presented independently the simplest member of this family.

The rest of this paper is organized as follows. In Section 2, the kinematics of the Tripteron are presented, and it is shown that the Tripteron has fully decoupled linear input-output equations, and no singularities within its workspace. Section 3 addresses some important design considerations concerning the Tripteron. It is shown that for a proper design, the workspace of the Tripteron is a rectangular parallelepiped. Some possible variations of the Tripteron architecture are also outlined. Then, the Tripteron is compared to other translational mechanisms. Finally, a practical prototype is shown and its mechanical design is discussed.

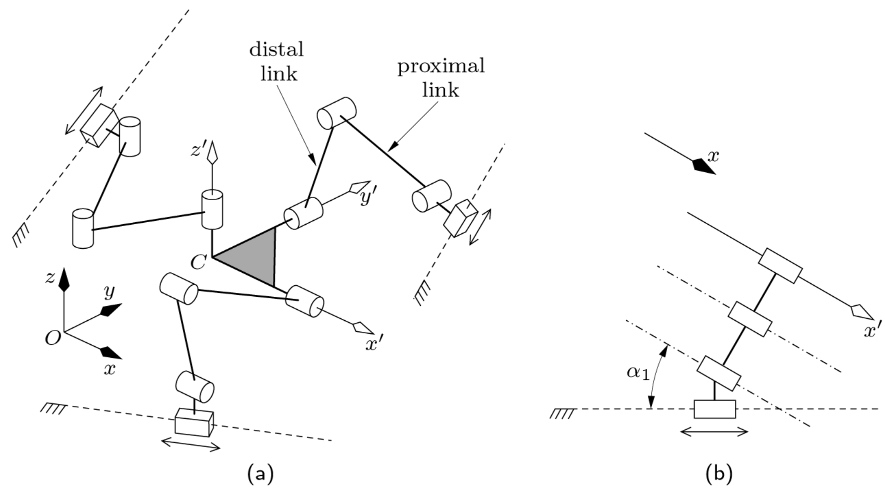

Fig. 1: *Schematics of (a) the Tripteron and (b) the projection of one of its legs onto a plane parallel to the axes of the leg's revolute joints and the direction of the leg's prismatic joint.*

2 Kinematics of the Tripteron

The Tripteron is a parallel mechanism with three legs, each being a 4-DOF serial mechanism whose links are connected by, in that order, an actuated prismatic joint fixed at the base, and three revolute joints whose axes are parallel to each other but not normal to the direction of the prismatic joint, as shown in Fig. 1(a). The terminal revolute joints of the three legs are connected to the mobile platform in such a manner that their axes are orthogonal. Although not necessary, let us assume for simplicity that these three axes intersect at one point, referred to as the center of the mobile platform and denoted by *C*. In leg *i*, let us denote the constant angle between the direction of the prismatic joint and any of the axes of the three revolute joints by α_i, as shown in Fig. 1(b), where $0 \le \alpha_i < \pi/2$ (in this paper, $i = 1, 2, 3$).

Let us denote by L_i, a line fixed to the base and parallel to the axes of the revolute joints in leg *i*. Obviously, line L_i will always remain parallel to the axes of the three revolute joints in leg *i*. Let us fix an orthogonal reference frame *Cx'y'z'* to the mobile platform in such a way that the *x', y'* and *z'* axes coincide with the axes of the terminal revolute joints of legs 1, 2, and 3, respectively. By the geometry of the legs, it follows that if the mechanism is assembled, then the axes *x', y'* and *z'* are parallel to the lines L_1, L_2, and L_3, respectively.

Now, imagine that all legs but leg i are disconnected from the mobile platform. If the actuator of leg *i* is blocked, then the platform can only translate in a plane normal to line L_i and rotate about axes parallel to L_i. Or from another point of view, the platform cannot translate in a direction that is not normal to line L_i nor rotate about axes that are not parallel to L_i. Back to the assembled parallel mechanism, it follows that if the other two actuators are blocked, then moving the *i*-th one, displaces the mobile platform along a line parallel to L_i. Since lines L_1, L_2, and L_3 are orthogonal, the Tripteron is a decoupled translational parallel mechanism.

Let us choose a base reference frame *Oxyz* with the same orientation as the mobile frame, such that point *C* coincides with point *O* when all three prismatic actuators have zero length, and increasing an actuator's length displaces the platform along the positive part of the corresponding base frame axis. Therefore, referring to Fig. 1(b), the following trivial system of decoupled linear equations governs the motion of the Tripteron:

$$x = k_1 \rho_1 \quad (1)$$
$$y = k_2 \rho_2 \quad (2)$$
$$z = k_3 \rho_3 \quad (3)$$

where $k_i = \cos\alpha_i$ is a reduction factor, $0 < k_i \leq 1$, ρ_i is the length of actuator *i* and *x, y* and *z* are the Cartesian coordinates of point *C* with respect to the base frame. These three equations represent the solution of the *direct kinematics* of the Tripteron.

Since $k_i \neq 0$, the above system of independent equations can be inverted to give the trivial solution to the *inverse kinematics* of the Tripteron. It should be noted, though, that for a given posture of the mobile platform, there are two possible configurations for each leg. However, the choice of configuration does not influence in any way the kinematic properties of the Tripteron. In fact, as we will mention later, the sequence of three revolute joints with parallel axes in each leg can be replaced with any other *n*-DOF planar chain ($n \geq 3$) such as a mechanism with *n* revolute joints with parallel axes. What counts only is the singularities of this planar mechanism, which will limit the workspace of the mechanism. Of course, in practice, what also counts is the link interference. Thus, strictly speaking, the system of independent linear equations (1–3) is valid only within the workspace of the Tripteron.

In the case of the original Tripteron, the condition for each leg is that the distance between the axes of the extreme revolute joints is smaller than the sum of the lengths of the *proximal* and *distal* links and larger than the absolute value of the difference between these two lengths.

Though simple as it is, the system of independent equations (1–3) can be rewritten in matrix form as

$$\rho = \mathbf{Jq}, \tag{4}$$

where $\mathbf{q} = [x,y,z]^T$ is the vector of output Cartesian coordinates, $\rho = [\rho_1, \rho_2, \rho_3]^T$ is the vector of input coordinates, and **J** is a constant diagonal matrix:

$$\mathbf{J} = \begin{bmatrix} \frac{1}{k_1} & 0 & 0 \\ 0 & \frac{1}{k_2} & 0 \\ 0 & 0 & \frac{1}{k_3} \end{bmatrix} \tag{5}$$

Differentiating Eq. (4), leads to:

$$\dot{\rho} = \mathbf{J}\dot{\mathbf{q}} \tag{6}$$

Hence, **J** is the Jacobian matrix of the Tripteron.

Since the Jacobian of the Tripteron is a constant diagonal matrix with nonzero elements, the mechanism does not have any singularities of Type 2. As for Type 1 singularities, they occur inevitably when the axes of the three revolute joints in a leg become coplanar. In fact, the Type 1 singularities are the boundaries of the workspace, not much different from the configurations when a prismatic actuator is at its limit.

Zlatanov et al. /16/ defined a different type of singularity that occurs in parallel mechanisms with constrained degrees of freedom, called a *constraint singularity*. In our case, this would be a singularity that allows the mobile platform to undergo (finite or infinite) rotations. However, it was shown at the beginning of this section that each leg i constrains any rotation of the mobile platform that is not parallel to line L_i. Since, these lines are orthogonal, the constraints never become dependent, meaning that the Tripteron has no constraint singularities. Therefore, the Tripteron *has no singularities inside its workspace*.

A translational parallel mechanism cannot be simpler than the Tripteron. Forget about all the problems concerning parallel mechanisms because they do not exist with the Tripteron. The Tripteron is simpler than the simplest serial mechanism. In fact, stacking three prismatic actuators in series will lead to a more complicated mechanism than the Tripteron, because it will not be decoupled in the Cartesian coordinates unless the directions of the prismatic joints are orthogonal. The Tripteron has trivial fully-decoupled inverse and direct kinematics, and no singularities within its workspace. Its kinematic performance is constant throughout its workspace. The simplicity of the Tripteron is attractive not that much because of the required small computation time (computer hardware nowadays is powerful enough) but because of the human factor. An operator will be more comfortable with a PKM whose principle is so intuitive. In fact, in the eyes of an operator and to the control algorithm, the Tripteron is basically identical to a standard XYZ stage.

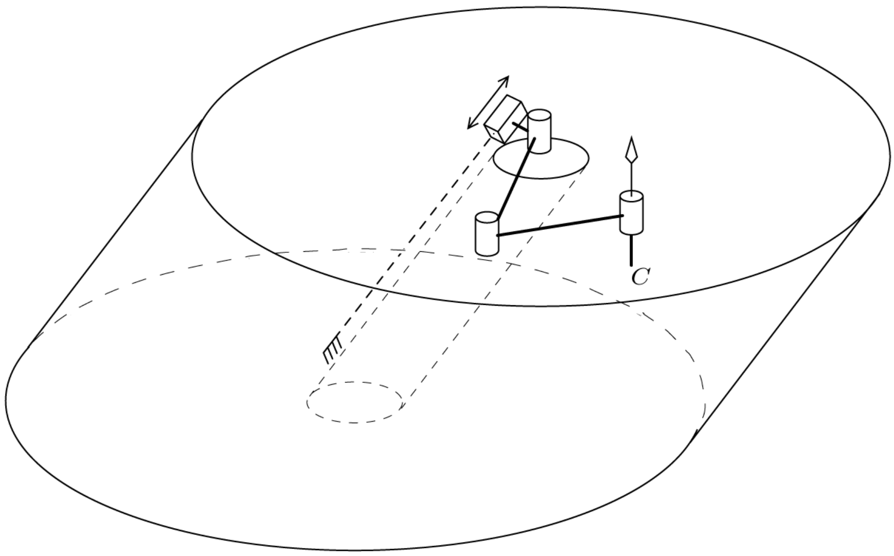

Fig. 2: *Schematic of the vertex space for one of the Tripteron's legs.*

3 Design considerations

One of great advantages concerning the Tripteron, other than its simplicity, is the variety of possible designs.

3.1 Possible geometries and workspace evaluation

The most influential design parameters of the Tripteron are the angles α_1, α_2, and α_3. These angles determine not only the three reduction factors (i.e., the output resolution) but also the overall shape of the mechanism.

Loyal to the geometric approach (see for example /17, 18, 19/), we will use it for computing the workspace of the Tripteron. Detaching all but leg *i* from the mobile platform, it can be easily seen that the so-called *vertex space* attainable by point *C* is the volume obtained by sweeping a circular annulus along the line segment associated with the prismatic joint of the leg (Fig. 2). The circular annulus is the workspace of the three-revolute planar mechanism in the leg. The two planar caps of the swept solid are normal to the line L_i, i.e, to the base *x, y* or *z* axis. The workspace of the Tripteron is the intersection of the three vertex spaces.

As researchers, the first thing on our mind was to implement in Matlab our geometric method in order to be able to optimize the workspace of the Tripteron by minimizing the lengths of the proximal and distal links in each leg. This could be done more promptly in a commercial CAD system, such as CATIA /18/. Figure 3(a) shows an example of the workspace of a Tripteron with relatively short legs.

We can obtain the best ratio between the lengths of the links and the volume of the workspace. A relatively large increase of the link lengths will result in only a negligible gain in the workspace volume. The problem is, however, that we just tried to market the Tripteron as the simplest spatial parallel mechanism and now we say that its workspace has the common complex shape of a parallel mechanism. How can we commercialize a pick-and-place robot or a PKM with such a complex workspace and claim that the device is simple? What is the use of having trivial kinematics, if at every step, we need to calculate whether we are still far from the workspace boundary?

Thus, our decision was to keep the links as long as it takes, so that the legs are never fully stretched or fully contracted. Of course, we still try to minimize their length, by carefully locating the prismatic actuators on the base and properly choosing the dimensions of the mobile platform. Geometrically, this means that the annular regions of the vertex spaces

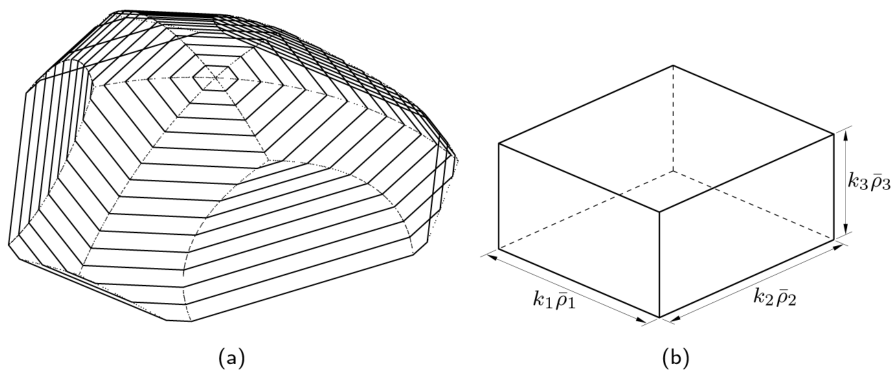

Fig. 3: *Examples of the workspace of the Tripteron for (a) relatively short legs (only the upper part is shown) and (b) relatively long legs.*

are relatively larger compared to the heights of the vertex spaces. Thus, when the three vertex spaces are intersected, it is the planar caps that limit the workspace and not the cylindrical surfaces. In other words, *the workspace of a Tripteron with sufficiently long legs is a rectangular parallelepiped* whose sides are of length $k_i\bar{\rho}_i$, where $\bar{\rho}_i$ is the stroke of actuator i, as shown in Fig. 3(b).

When $\alpha_1 = \alpha_2 = \alpha_3$, the three reduction factors are equal, as are the elements of the diagonal Jacobian matrix, and the resulting mechanism is said to be *isotropic*. Two obvious designs of the Tripteron are the one shown in Fig. 4(a), in which the directions of all prismatic joints are parallel and $\alpha_1 = \alpha_2 = \alpha_3 = \cos^{-1}\sqrt{1/3}$, and the one shown in Fig. 4(b), in which directions of all prismatic joints are coplanar and making equal angles and $\alpha_1 = \alpha_2 = \alpha_3 = \cos^{-1}\sqrt{2/3}$. But of course, the most obvious design is the one in which the direction of the prismatic joint is parallel to the axes of the revolute joints, for every leg, i.e., $\alpha_1 = \alpha_2 = \alpha_3 = 0$ (Fig. 5). The latter, having no reduction factors ($k_1 = k_2 = k_3 = 1$) is basically the "parallel twin" of the well known serial Cartesian robot or XYZ stage.

Clearly, the orthogonal version of the Tripteron ($k_1 = k_2 = k_3 = 1$) is the most appealing one. But the other versions are interesting too. Firstly, they provide a mechanical reduction factor, which increases the output resolution for the same actuators (at the price of a smaller workspace). And secondly, they provide for a more compact design. For example, in some applications, it might be useful to attach all three motors on a common plane. Finally, we may also use different α's for the different legs. For example, it might be appropriate to design only the z leg with a zero α_3, i.e., $k_3 = 1$, to achieve faster (though less precise) motion in the z direction.

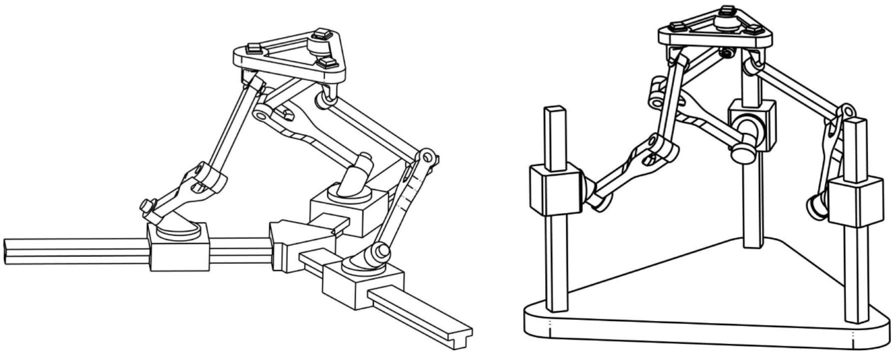

Fig. 4: *Two versions of the Tripteron.*

3.2 Possible variations of the architecture

The Tripteron is an *overconstrained mechanism*. Each leg constrains not one but two rotations. This leads to an increased complexity in the determination of the reaction forces. Various authors have expressed concerns with such mechanisms since geometric errors in the mechanism may lead to internal antagonistic forces. To avoid this, we could insert a fourth revolute joint in each leg, with an axis that is not parallel to the axes of the other three revolute joints /20/. For example, we could replace the revolute joints at the mobile platform by universal joints. Kinematically, the mechanism remains unchanged and the new joints remain theoretically inactive. Inserting such inactive joints, however, increases the structural complexity and the clearances of the mechanism. In contrast, an overconstrained mechanism usually has a higher stiffness than its kinematically equivalent non-overconstrained (isostatic) counterpart.

Finally, we should restate that the basic principle of the Tripteron remains unchanged if we replace the three-revolute chain in each leg by any nDOF planar chain ($n \geq 3$).

3.3 Comparison with other architectures

One of the most popular designs for translational parallel mechanisms is the Delta robot design /1/. The latter has three identical legs, each consisting of a revolute or a prismatic actuator fixed at the base, a revolute joint, a complex parallelogram, and another revolute joint. The structural design of the Delta robot is obviously more complicated than that of the Tripteron as is the inverse and direct kinematics, and the workspace shape.

In the original rotary actuation version, chosen for industrial pick-and-place applications, the Delta robot has a clear advantage in terms of speed, due to the *amplifying lever effect*. In the Tripteron, we saw that the velocities are directly proportional to the velocities of the actuators by a factor that is smaller or equal to 1. Thus, the Tripteron is only as fast as its actuators, or slower.

In the linear actuation version, chosen for PKM applications, the Delta robot is not necessarily faster than the Tripteron, given the same prismatic actuators. However, the output resolution of the Delta robot varies throughout the workspace, while it is constant for the Tripteron. The only advantage of the Delta robot design is that its links are subject to tensile-compressive stresses only, while the links of the Tripteron are subject to compound stresses, including bending. Therefore, the Tripteron is an interesting alternative for high-speed PKM applications where the loads are less important. Another application where the Tripteron could be a winning alternative is micropositioning /22, 23/, where flexures may be used instead of rigid joint. We are currently building such a micro-Tripteron with piezo-actuators.

But, we have no interest in comparing the Tripteron to other translational parallel robots or PKMs. We rather challenge the serial Cartesian robot (or XYZ stage). Because the Tripteron *is basically an improved version* of the serial Cartesian robot. There is no doubt, that at the price of a few more links and bearings, the Tripteron should be faster and more precise than the serial Cartesian robot that has to carry its own heavy actuators. And if, after all these years, there are just over a thousand translational parallel robots or PKMs on the market, there are certainly more that 15,000 serial Cartesian robots sold... each year /21/.

4 Building our first prototype

We chose to build the most intuitive version of the Tripteron, namely the one with $\alpha_1 = \alpha_2 = \alpha_3 = 0$ (Fig. 5). Since the reduction factors for this version are all equal to 1, the mechanism behaves exactly like a serial Cartesian robot. As for the possible variations of the architecture, in our experience, we have never had problems with even the most overconstrained mechanisms such as our 3-DOF spherical parallel mechanism called the Agile eye /24/. Therefore, we decided to build the original overconstrained version and do not regret this decision – the prototype moves quite smoothly.

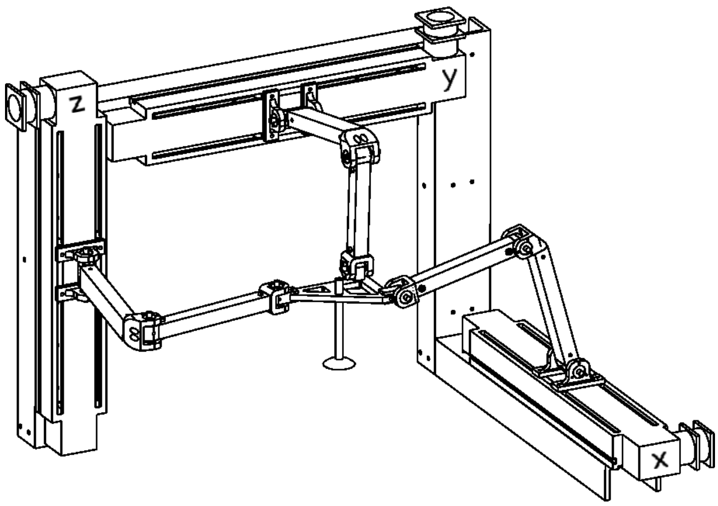

Fig. 5: *A CAD model of the Orthogonal Tripteron.*

As one can see in Figs. 5 and 6, the linear actuators were placed on the base in such a manner as to occupy a minimum space and to allow other equipment such as a conveyor to run under the robot. Similarly, the leg configurations were chosen so that they do not interfere with the space that is under the end-effector. It should be noted that the mobile platform was designed in order to avoid leg interference and minimize the necessary lengths of the links. Thus, the axes of the revolute joints on the platform do not intersect at one point. This means simply that an offset constant should be added to each of equations (1–3). As for the links, they were chosen to be as long as necessary to allow the mechanism to have its maximum rectangular parallelepiped workspace. Furthermore, the Orthogonal Tripteron was designed in such a way as to avoid any link interference.

Standard thin-wall aluminium extrusion rectangular tubes were selected for the proximal and distal links. The proximal links have an outer section of 38.10 × 25.40 mm (1.5 x 1 inch) and a wall thickness of 3.18 mm (1/8 inch). The distal links have an outer section of 25.40 × 25.40 mm (1 × 1 inch) and a wall thickness of 1.59 mm (1/16 inch). The lengths of each link are given in Table 1 (the actuators directions are shown in Fig. 5). Note that the lengths of the proximal links are not the lengths of the rectangular tubes but the distances between the revolute-joint axes.

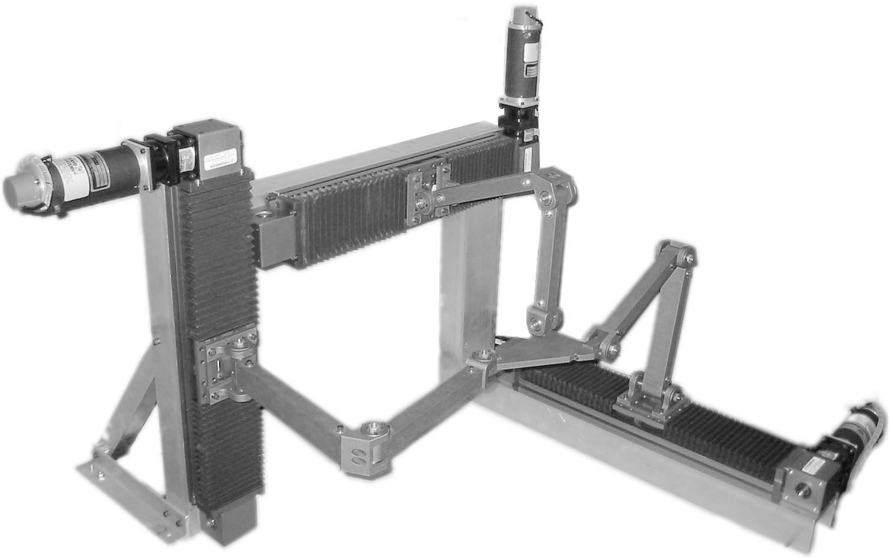

Fig. 6: The prototype of the Orthogonal Tripteron.

Table 1: Link lengths.

actuator direction	proximal link [mm]	distal link [mm]
x	247.51	223.00
y	237.57	203.00
z	272.37	258.00

A finite element structural analysis was performed with Pro/Engineer's Pro/Mecanica to see how the *z* axis leg will react to a load placed at the end-effector. Typical results give a deflection of about 0.5 mm for a load of 25 N. These results are satisfactory since the payload of the prototype is intended to be less than a kilogram.

The most important question was, however, the choice of actuators. Indeed, once leg interference is avoided through proper design, the Tripteron is basically as good as its actuators, if we ignore the relatively small mass of the mobile structure. The first solution was to use ball screw or belt-driven linear slide systems. The second one was to use linear servo motors. The big advantage of linear servo motors is that there are no mechanical elements between the motor and the linear output, resulting in better accuracy, acceleration, and speed. In the traditional ball screw or belt-driven linear slide system, there is gear backlash, ball screw speed limit, and belt elasticity, to name a few disadvantages. All of these factors decrease the performance of the system. But since the linear servo motors are relatively new on the market, they are still far too expensive, compared to their traditional counterparts. Besides, we wanted our prototype to be in fair competition with typical serial Cartesian robots, and hence, have the same actuators.

Therefore, we chose two Thomson Accuslide 2HE-M10-OZP-B-L525m belt-driven linear slides for the *x*- and *y*-axes, and one Thomson Accuslide 2HB-M10-OYP-H-L525m ball screw linear slide for the *z* axis. The *z* actuator was chosen different from the other two because it has to support permanently the weight of the mobile part of the mechanism. (The *z* actuator in Fig. 6 is a belt-driven linear slide, which was later replaced with a ball screw slide.) The effective stroke for all actuators (with bellows) is about 250 mm. Thus, the workspace of our Orthogonal Tripteron is a cube of side 250 mm.

The mass of the mobile part of our Tripteron prototype is less than 3.5 kg. Only one of the Thomson linear slides, the lighter beltdriven one, on top of that, is more than 6.5 kg. In a serial Cartesian robot with the same actuators, the first actuator will have to move the second and third one, or more than 13 kg. Thus, it is obvious that our Tripteron should be faster than an equivalent serial Cartesian robot. Unfortunately, we are still unable to match the advertised performance of commercial serial Cartesian robots.

Currently, our prototype is driven by 3 DC brush-type servo motors having a continuous stall torque of 0.388 Nm and a maximum operating speed of 6000 RPM. The control of the prototype consists of a simple PID loop on each DC motor. The control is implemented in Simulink and executed on RT-LAB's realtime platform by Opal-RT. The prototype was tested and works well, but is still slower than commercial Cartesian robots. For the *x*- and *y*-axes, we obtain a speed of 0.7 m/s and an acceleration of 4 m/s^2, and for the z-axis, the speed is about 0.4 m/s and the acceleration is about 2 m/s^2 . The z-axis speed is acceptable since the speed limit of the slide is 0.5 m/s, but for the x- and y-axis, the obtained speed is well below the predicted one of 1.87 m/s.

Indeed, we have not yet explored the full potential of the linear guides, since we currently use DC motors. Brushless servo motor having much better characteristics were also purchased and will be eventually installed. With these motors, the performance of our prototype is expected to be substantially improved.

In spite of our difficulties in beating, in lab conditions, the performance of commercially available serial Cartesian robots, we were still able to demonstrate the viability of the Tripteron concept. A manufacturer of such serial Cartesian robots could undoubtedly improve their performance by adopting the Tripteron concept.

5 Conclusion

A novel revolutionary-simple 3-DOF translational parallel mechanism, named Tripteron, was described in this paper. Because of the common prejudice towards parallel robots, we had to fill in pages of explanations that there are no singularities, that the inverse and direct kinematics are trivial, that the workspace is a rectangular parallelepiped, and so on. But there is only one thing to remember – a well-designed Tripteron is basically an improved Cartesian robot.

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